Turn in the following problems:

1. Find the equations of all tangent lines to the curve $x^2 + 4y^2 = 8$ that pass through the point $(-4, 0)$.

2. Water is leaking out of an inverted conical tank at a rate of 1.5 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 10 cm and the diameter at the top is 6 cm. If the water level is rising at a rate of 1.0 cm/min when the height of the water is 2 cm, find the rate at which water is being pumped into the tank.

   (a) Draw a picture of the situation for any time t.

   (b) What quantities are given in the problem? What is the unknown quantity you are looking for?

   (c) Write an equation that relates the quantities.

   (d) Finish solving the problem.

3. A police helicopter is flying at 150 mph at a constant altitude of 0.5 mile above a straight road. The pilot uses radar to determine that an oncoming car is at a distance of exactly 1 mile from the helicopter, and that this distance is decreasing at 190 mph. Find the speed of the car.

   (a) Draw a picture of the situation for any time t.

   (b) What quantities are given in the problem? What is the unknown quantity you are looking for?

   (c) Write an equation that relates the quantities.

   (d) Finish solving the problem.

4. Compute the derivatives for the following functions. Be sure to show all your work.

   (a) $C(q) = \sec\left(eq^2 + \pi\right)$

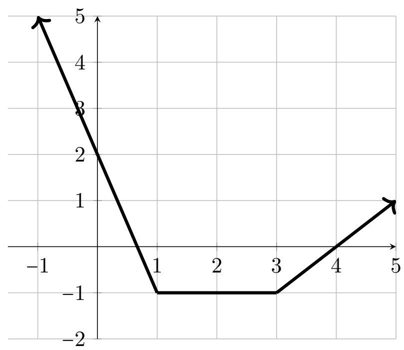
   (b) $s(t) = \dfrac{3}{\sqrt[5]{t^2 - 3\cos(t) - 1}}$

   (c) $h(x) = (3x^2 - x) \cdot \dfrac{(x-2)}{(2+x)}$

5. If $xy + e^y = e$, find the value of y'' at the point where $x = 0$.

6. A table of the functions $f(x)$ and $f'(x)$ and a graph of the piecewise linear function $g(x)$ are shown below.

x	$f(x)$	$f'(x)$
-1	11	-7
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	2



$y = g(x)$

(a) Given $h(x) = f(g(x))$, find $h'(0)$.

(b) Given $p(x) = (g(x))^3$, find $p'(2)$.

(c) Given $k(x) = g(f(x))^{2/3}$, find $k'(1)$.

These problems will not be collected, but you are expected to solve. You might need the solutions during the semester:

7. Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

where $p(t)$ is the proportion of the population that knows the rumor at time t and a and k are positive constants. (In Calculus 2 we will see that this is a reasonable equation for $p(t)$.)

   (a) Find $\lim_{t\to\infty} p(t)$

   (b) Find the rate of spread of the rumor.

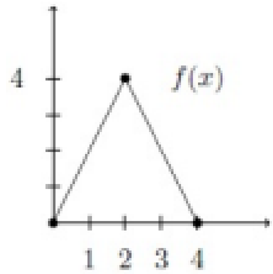
   (c) Graph p for the case $a = 10$, $k = 0.5$ with t measured in hours. use the graph to estimate how long it will take for 80% of the population to hear the rumor.

8. Two curves are **orthogonal** if their tangent lines are perpendicular at each point of intersection. Show that the given families of curves are **orthogonal trajectories** of each other, that is, every curve in one family is orthogonal to every curve in the other family. Sketch both families of curves on the same axes.

$$y = ax^3 \text{ and } x^2 + 3y^2 = b$$

9. A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

10. The graph of $f(x)$ is shown and the table gives values of $g(x)$ and $g'(x)$.



x	0	1	2	3
$g(x)$	4	3	2	1
$g'(x)$	-1.1	-0.9	-1.2	-0.8

(The function $f(x)$ is piecewise linear)

(a) Given $h(x) = f(g(x))$, find $h'(1)$.

(b) Given $k(x) = g(f(x))$, find $k'(3)$.

(c) Given $l(x) = g(g(x))$, find $l'(2)$.

(d) Given $m(x) = \sqrt{f(x)}$, find $m'(1)$.

Optional Challenge Problems

Sketch the circles $x^2 + y^2 = 1$ and $(x - 3)^2 + y^2 = 4$. There is a line with positive slope that is tangent to both circles. Determine the points at which this tangent line touches the circle.